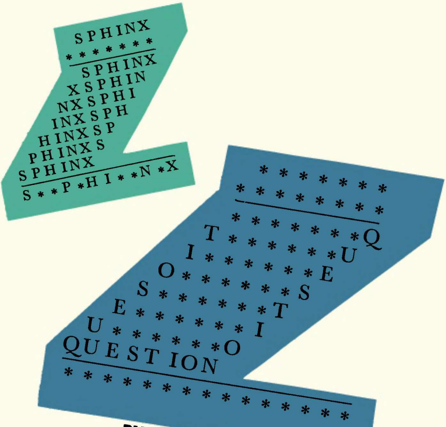
150
Puzzles In Crypt-Arithmetic



BY MAXEY BROOKE

150 PUZZLES IN CRYPT-ARITHMETIC

by

Maxey Brooke

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PUZZLES

PUZZLES

1. The word "crypt-arithmetic" was first introduced by M. Vatriquant writing under the pseudonym "Minos." In the May, 1931, issue of *Sphinx*, a Belgian magazine of recreational mathematics, he proposed this problem with these remarks:

"Cryptographers, to hide the meaning of messages, put figures in places of letters. By way of reprisal, we will replace each digit of the following problem with a distinct letter."

$$
\begin{array}{r}
A\ B\ C \\
\times\ D\ E \\
\hline
F\ E\ C \\
D\ E\ C\ \\
\hline
H\ G\ B\ C
\end{array}
$$

2. Although the word was new, the type of puzzle was older. The earliest one in my collection is from the *Strand Magazine* for July, 1924:

$$\text{TWO} \times \text{TWO} = \text{THREE}$$

3. If the readers of *Sphinx* did not originate the problem, they certainly elevated it to a plane never achieved before or since. Most of the problems in this book originated in that magazine. But just to assure you that the art has not died out, here is a recent one from *Mathematics Magazine* for May, 1960:

```
E|R T M P
   -------
  L|B T O
     -----
    I|T S
      ---
      R L
```

4. An earlier ancestor of crypt-arithmetic is the type of problem known as "Arithmetical Restorations," which were probably invented in India during the Middle Ages. An early problem of this type was proposed by W. E. H. Berwick in the *School World* for July, 1906. All the digits except the seven 7's have been replaced by dots. The solution is unique.

```
....7.)..7.......(..7..
       ......
       .....7.
       .......
         .7....
         .7....
         .......
         ....7..
            ......
            ......
```

But enough history. There are no mathematical formulas to solve these problems. It is a matter of pure ingenuity, reasoning power, and perseverance. However, a few general rules might be given.

Take addition and subtraction. When the numbers at the right of a sum are of the form $\begin{array}{r} A \\ +A \\ \hline A \end{array}$ or $\begin{array}{r} B \\ +A \\ \hline B \end{array}$, A must be equal to zero.

But if in the body of a problem we find $\begin{array}{r} xxAx \\ +xxAx \\ \hline xxAx \end{array}$ or $\begin{array}{r} xxBx, \\ +xxAx, \\ \hline xxBx \end{array}$ A may be equal to zero or 9.

The problem $\begin{array}{r} CCA \\ +\ BBA \\ \hline *CAA \end{array}$ can be analyzed thus: A = 0. B can equal 0 or 9, but since A = 0, B must equal 9. C now can equal only 1 because B+C ends in zero. The problem becomes

$$110+990 = 1100$$

When two numbers of n digits give a total of $n+1$ digits, the left digit of the sum must be 1.

$$\begin{array}{r} AA \\ +\ BB \\ \hline CBC \end{array}$$

C can only equal 1. A+B ends in B, so A equals 0 or 9. But A cannot be zero because it is the first digit of a number, hence A = 9. Now, A+B = 9+B = 11. B = 2 and the addition becomes 99+22 = 121.

Now let us analyze a problem that is so familiar that it has almost achieved the status of a chestnut. It concerns a telegram received by the parents who had a son in college:

$$\begin{array}{r} SEND \\ +MORE \\ \hline MONEY \end{array}$$

We see at once that M in the total must be 1. M+S gives a two-digit number, so S must be either 8 or 9. In either case O will be zero or 1, and since M is 1, O must equal zero.

If O is zero, E+O cannot be 10, so there is no 1 to carry over to the column S+M. This means S = 9.

Since E+O = N and O is zero, N must be 1 greater than E and N+R must total more than 10. These can be expressed in the equations:

$$E+1 = N$$

$$N+R+(+1) = E+10$$

We use (+1) because we do not know whether or not 1 is carried over from the column D+E.

Subtract the first equation from the second:

$$R+(+1) = 9$$

But R cannot equal 9 because S does, hence R must equal 8 and 1 is carried over from the column D+E.

D+E must total at least 12, since Y cannot be 0 or 1. The only digits we have left are 7+6 and 7+5. One of these must be E, and N is 1 greater than E. Hence E must be 5, N must be 6, and D = 7. This makes Y = 2 and the problem is solved.

```
   9567
+  1085
  10652
```

Or take a multiplication problem like this:

```
      A L E
      R U M
    W I N E
   W U W L
  E W W E
  E R M P N E
```

Any number multiplied by zero is zero; any number multiplied by 1 is the number itself. These two digits are usually the easiest to determine. For example, in this problem N + L = N, hence L = zero.

R, U, and M can be eliminated as 1 because none of them produces ALE when multiplied by ALE. E is not 1 since U × E does not give U.

The second partial product ends in L which is zero. Thus

	0	1	2	3	4	5	6	7	8	9
A	×					×				
L	■	×	×	×	×	×	×	×	×	×
E	×	×	×	×	×	■	×	×	×	×
R	×	×	×		×	×	×		×	
U	×	×		×		×		×		×
M	×	×	×		×	×	×		×	
W	×					×				
I	×					×				
N	×					×				
P	×					×				

either U or E is 5. And since $R \times E$ and $M \times E$ both end in E, E must be 5. This makes R and M odd numbers and U an even number.

An array like the one above is cumbersome but it is a convenient way of keeping track of where we are. An × means that a number cannot be a given letter or a letter cannot be a given number. When a letter and number are matched, that square is blacked in.

Since U is 2, 4, 6, or 8, look at the second partial product. $U \times LE = U \times 05 = WL$. If U is 2, W is 1; if U is 4, W is 2; if U is 6, W is 3; and if U is 8, W is 4. This further narrows our search.

R enters into the picture in a similar manner:

If U is	and W is	then R is	
2	1	3	
4	2	5	(but E is 5)
6	3	7	
8	4	9	

Or $R = 2W+1$.

Referring to the second and third columns from the left, we see that $2W+U$ must be greater than 9. Thus the first two sets of values are eliminated. Our array now looks like this:

	0	1	2	3	4	5	6	7	8	9
A	×					×				
L	■	×	×	×	×	×	×	×	×	×
E	×	×	×	×	×	■	×	×	×	×
R	×	×	×	×	×	×	×		×	
U	×	×	×	×	×	×		×		×
M	×	×	×		×	×	×		×	
W	×	×	×			×	×	×	×	×
I	×					×				
N	×					×				
P	×					×				

M is odd and 2W+U+1 = M. If W = 3 and U = 6, then M = 3, which is impossible. The only remaining combination is for W = 4 and U = 8, in which case M = 7. The array now looks like this:

	0	1	2	3	4	5	6	7	8	9
A	×				×	×		×	×	
L	▨	×	×	×	×	×	×	×	×	×
E	×	×	×	×	×	▨	×	×	×	×
R	×	×	×	×	×	×	×	×	×	
U	×	×	×	×	×	×	×	×	▨	×
M	×	×	×	×	×	×	×	▨	×	×
W	×	×	×	×	▨	×	×	×	×	×
I	×				×	×		×	×	
N	×				×	×		×	×	
P	×				×	×		×	×	

We see by inspection that R = 9. We now have enough of the digits so that the others can be obtained simply:

```
      A 0 5
      9 8 7
    -------
    4 I N 5
  4 8 4 0
5 4 4 5
-----------
5 9 7 P N 5
```

And A = 6, I = 2, N = 3, and P = 1.

The next operation to consider is the extraction of square root.

```
√*****A = AA*
 **
 --
 ****
 ****
 ----
  *****
  *****
  -----
      0
```

This is a perfect square and can end only in 0, 1, 4, 5, 6, or 9, one of which must equal A.

The first partial product is the square of A. Since it consists of two digits, the possibilities of $A = 0$ and $A = 1$ are eliminated.

The second partial product is formed by multiplying $(2A + 10 + A)$ by A, or $21A^2$. There are four digits in this product; therefore $21A^2$ is greater than 1000, or A^2 is greater than 47. The only possibility is $A = 9$.

The last digit of the root must be 3 or 7 if the last digit of the square is 9. Assume it to be 3. The third partial product would be 1983×3. This is impossible since the product contains five digits. By elimination, the last digit is 7.

The square root is 997.

Because of custom, most of our problems are expressed to the base 10. This is not at all necessary. Some of our better cryptarithms are to other number bases. The one below is to the base 8:

```
          L O T I
          R A V I
          -------
        * * * T T
        L * * *
      * * * * I
    0 * * * *
    -------------
    L * * * E * 0 *
```

First, let's set up a multiplication table to the base 8:*

	1	2	3	4	5	6	7	10
1	1	2	3	4	5	6	7	10
2	2	4	6	10	12	14	16	20
3	3	6	11	14	17	22	25	30
4	4	10	14	20	24	30	34	40
5	5	12	17	24	31	36	43	50
6	6	14	22	30	36	44	52	60
7	7	16	25	34	43	52	61	70
10	10	20	30	40	50	60	70	100

* This is different from the usual multiplication table in that 8 and 9 are missing. It is the kind of table that might have developed in Disneyland where the cartoon characters have only four fingers on each hand.

In the 10 system, every tenth number repeats:

By inspection, V = 1.

The product I × I terminates in T, so I must be 2, 4, or 6. All other squares end in 1.

Since A × I ends in I, and since TI × I ends in TT, the possibilities are:

V	I	T	A	TI × I = TT	
1	2	4	5	04	(impossible)
1	4	0	3, 5, or 7	20	(impossible)
1	6	4	5	44	

Thus I = 6, T = 4, and A = 5.

L = O + 1, so O = 2 and L = 3, the only choice we have left. And by default, E must be zero.

To make the problem more interesting, the letters line up to spell a famous name:

1	2	3	4	5	6	7	0
V	O	L	T	A	I	R	E

A final problem is the Fifth of November cryptarithm. It requires no figures but is solved by sheer logic.

```
                VENT
            NOVEMBRE
            --------
               N***E
              M***R
             O***B
            S***M
           N***E
          E***V
         R***0
        B***N
        -------------
        *************
            T = 1
```

0, 1, 2, 3, 4, 5, 6, 7, 8, 9
10, 11, 12, 13, 14, 15, 16, 17, 18, 19
20, 21, 22, 23, 24, 25, 26, 27, 28, 29

In the 8 system, every eighth number repeats:

0, 1, 2, 3, 4, 5, 6, 7
10, 11, 12, 13, 14, 15, 16, 17
20, 21, 22, 23, 24, 25, 26, 27

In both systems 2 × 2 = 4 and 2 × 3 = 6, but 2 × 4 = 10 in the 8 system and 2 × 5 = 12.

All the partial products are one digit longer than the multiplicand. The first digit of each product is smaller than the corresponding digit in the multiplier. Thus

$$S < M < R < O < B < N < E < V$$

So the values must be, respectively, 2, 3, 4, 5, 6, 7, 8, 9.

5. Three letters:

```
      A B C
      B A C
     ------
    * * * *
    * * A
* * * B
-----------
* * * * * *
```

6. Four letters:

```
        A B C D
        A B C D
        -------
      * * A D C
    * * * * *
  * * * * C
* * * * *
---------------
* * * * * * * *
```

7. And five letters:

```
        A B C D E
        C E D B A
        ---------
        * * * * *
    * * * * * *
  * * * * * *
* * * * * D
* * * * B
-----------------
* * * * * * * * *
```

8. English translation: "Who can solve this?"

```
QUI )TROUVE (CECI
     ***
     ---
      ***
      ***
      ---
       ***
       **E
       ---
        ***
        ***
        ---
          0
```

9. Again, five letters:

```
      ABCDE
        FGA
     ------
     **H***
     BB***
   **H***
   --------
   *F***D**
```

10. Four A's:

```
        AA*
   √******A
    **
    --
     ***
     ***
     ---
      ****
      A***
      ----
```

11. Easter (Pâques) comes on April 16 some years:

```
                 PLS
        -------------
AVRIL ) 1**6*****
         AVRIL
         -----
         ******
         ****16
         ------
          ******
          PAQUES
          ------
               0
```

12. In French it means March; in English it stands for war. In either language it's cryptic:

```
          A R * *
          R A * *
          -------
        * * * * M
      * * * * A
    * * * * R
    M A R S
    ---------------
    * * * * * * * *
```

13. Continuing with the months, here is February:

```
          * 6 * * * *
       ---------------
   2 8 ) F E V R I E R
         * *
         ---
         * * *
         S I X
         -----
           * * *
           * * *
           -----
             * * *
             * * *
             -----
                 0
```

14. A five-letter multiplication:

```
              A B C D E
              C E D B A
              ---------
              * * * * *
            * * * * F
          * * * * * G
        * * * * * D
      * * * * * A
      -------------------
      * * * * * * * * * *
```

15. Literally, the "codfish suitcase":

```
          V A L I S E
            M O R U E
        -------------
          * * * 1 6 1 6
        * * * * * * *
      * * * * * * *
    * * * * * * *
  * * * 3 2 3 2
  -------------------
  * * * * * 1 6 1 6 1 6
```

16. In this QUESTION, the sums of the digits of the multiplier and the multiplicand are equal:

```
                  * * * * * * *
                * * * * * * * *
                ---------------
                * * * * * * * Q
              T * * * * * * U
            I * * * * * * E
          O * * * * * * S
        S * * * * * * T
      E * * * * * * I
    U * * * * * * O
  Q U E S T I O N
  ---------------------------
  * * * * * * * * * * * * * * *
```

17. This division problem comes from the *Strand Magazine* for 1930:

```
          *******
       ----------
    ***)5********
        ***
        ---
         ****
          ***
          ---
           5**
           ***
           ---
           *5**
           ****
           ----
              0
```

18. M. Pigeolet was the crypt-arithmetic editor for *Sphinx* for many years and a famous crypt-arithmetician in his own right. This problem was devised in his honor. The answer is a cube and all the T's are given.

```
      E * * * T
          * * E T
    -----------
    * * * * E T
    T * * * *
E * * * * *
E * * * T
---------------
P I G E O L E T
```

19. Problem for September:

```
  S E P T E M B R E         S E P T E M B R E
- E R B M E T P E S       + E R B M E T P E S
-------------------       -------------------
  M P P B R P S S M         B * R B M R R E B
```

20. The depression was no joke, but this joke came out of the depression:

USA+FDR = NRA
USA+NRA = TAX

21. Double multiplication:

```
      D
    × B
    ---
    * D
  × * C
  -----
  A C *
* * *
-------
A * C D
```

22. DEF and FAE are multiples of 11. Find the number D***A***F, which is a cube, if

```
    D E F
    F A E
    * H F
  -------
  * * H D
```

23. A positive and negative multiplied together do not necessarily cancel each other:

```
              N E G A T I F
              P O S I T I F
              -------------
              * * * * * * *
            * * * * * * *
          * * * * * * *
        * * * * * * *
      * * * * * * *
  * * * * * * * *
  * * * * * * *
---------------------------
N * * * * * * * E G A T I F
```

24. Here is a Slavic dance. All the letters are prime numbers.

```
          V A L S E
          S L A V E
          ---------
        * * * * * *
      * * * * * *
    * * * * * *
    * * * * *
  * * * * *
  -----------------
  * * * * * * * L E
```

25. No letters at all:

```
***)******(****.****
    ***
    ---
      ***
      ***
      ---
       ***
       ***
       ---
        ***
        ***
        ---
          ****
          ****
          ----
```

26. RG and RA are two consecutive numbers. Their squares are OHEM and OMLI. The sum of the numbers is LET and the sum of the squares is RTOR. Find the key.

27.

$$\text{ALGER} = \text{R}^2 \times \text{NIG}$$
$$\text{DELTA} = \text{R}[(\text{AADR} \times \text{O}) + \text{R}]$$

R is prime.

28. ADDD, AACA, BCDB, and BDAC are four prime numbers. What are they?

29. There are 24 possible combinations of the four digits ABCD and these combinations include:

4 prime numbers
7 products of two primes
1 square of a prime
8 numbers divisible by 2
2 numbers divisible by 4
1 number divisible by 8
1 number divisible by 16

What are they?

30. ABC is a prime number and *AC*BC is its square.

31. I found part of a problem in division. The right-hand side of the paper, including the answer, was unfortunately torn off. But you can still reconstruct what is left:

```
ABC )DEF
      JAC
      ---
      GJF
      GHD
      ---
        K
        B
```

32. What number represents the word ODER if ODER = 18 (DO + OR)?

33. A number is the product of two prime numbers. When it is divided by each of its factors, we obtain:

```
         xxxx                   xxxx
     ________               ________
xxxx)xxxxxxxx          xxxx)xxxxxxxx
     xxxx                   xxxx
     ----                   ----
     xxxxx                    xxxx
      xxxx                    xxxx
      ----                    ----
       xxxxx                    xxxx
       xxxxx                    xxxx
       -----                    ----
```

34.

MA + SU + RE = OIL
MA + US + ER = LOI

35. Chemistry lesson:

ACIDE + ETHER = ALCOOL

LIT in reverse is contained in REEL

36. ABCBCACAB = DB × DD × DE × FD × HD

37. PLIC × PLOC = PL**UIE

38. AABAC can be decomposed into the prime factors $A^3 \times B \times C^2 \times D^2$.

39. In setting up the product of two powers:

$$a^b \cdot c^a$$

the printer accidentally composed a number of four digits:

$$a\ b\ c\ a$$

Since the two have the same value, perhaps you can find them.

40. The Sphinx problem:

```
          S P H I N X
        * * * * * * *
        -------------
          S P H I N X
        X S P H I N
      N X S P H I
    I N X S P H
  H I N X S P
P H I N X S
S P H I N X
-----------------------
S * * P * H I * * N * X
```

41. Double-barreled multiplication (up and down):

```
        * * * * * * * *
        ---------------
              * * * * *
            * * * * *
          * * * *
        * * * *
        -------
        H S I X
        O I Z H
        -------
        * * * X
    * * * * *
    * * * X
  * * * *
---------------
* * * * * * * *
```

42. The arithmetic mean of NED and SASH is SHUN. Their geometric mean is SEND, and their harmonic mean is SEED.

43. REDACTIONS × C = TTTTTTTTSTS

44. In this magic checkerboard, the sum of the numbers in each horizontal row and vertical column is constant and equal to CEL × AA. The 32 numbers form an arithmetic progression.

GUR		CFR		AEA		GEO	
	LIO		FOA		FCR		OUR
ARI		URE		VAL		LLF	
	IIC		OGG		RAI		RUE
FGI		LCE		AUL		UFF	
	RRL		IEF		ILO		OFA
AAG		GLC		GRU		COU	
	IUU		OCU		LGC		FIG

45. $(\text{ANNE})_{\text{base } 8} - (\text{ANNE})_{\text{base } 5} = (\text{ANNE})_{\text{base } 7}$

46. Even the calculus is not exempt:

$$\int_A^B (x-1)\,dx = AA + A$$

47. CHINE + ASIE = JAPON

AS is a cube, JA and JAP are squares.

48.

```
          N U T A C
          E R I Q O
          ---------
          * * * * *
      * * * * * *
    * * * * * *
  * * * * * *
  -------------------
  N A R C O T I Q U E
```

49.

```
   A B C D E
 + E D C B A
   ---------
   F G H K L
```

AB is double a prime number.

50.

```
        * * * * * *
            * * * *
        -----------
        * A * * * *
    * * * * * *
  A A A A A A
  -------------------
  * * * * * * * * *
```

51.

$$\frac{\text{TERRE}}{\text{RAYON}} = 2\pi$$

52.

```
         G E F
 3/-----------------
 √ A B B C B D E D F        3 × G² = G K         3 × (G E)² = L B C D
     F G                    ------------         ----------------------
     ---                      G K C C                L B C D C C
    H K C B D                 A C K C                    K K B C
    H D F G E                     K A                        D F
    ---------                 -------                -----------
      G D L G E D F           H E F A                L B E A H F
      G D L G E D F               × E                        × F
      -------------           -------              -------------
                            H D F G E              G D L G E D F
```

53.

```
               2xxxx2
       --------------
xxxxxx)222222222222
       xxxxxx2
       -------
        x2x2xx2
        xxxxxxx
        -------
         xxxxxx2
         xxxxxxx
         -------
          xxxxxx2
          xxxxxxx
          -------
           xxxxxx2
           xxxxxxx
           -------
            xxxxxx2
            xxxxxxx
            -------
                  0
```

54.

```
       x7xxx
    --------
xxx)xxxxxxxx
    xxxx
    ----
      xxx
      xxx
      ---
      xxxx
       xxx
      ----
        xxxx
        xxxx
        ----
```

55. A printer, setting up a problem in division, found that he had no type numbers except 0. So he substituted X for all the other numbers. Can you work out the problem?

```
                 XXXXXX
        ___________________
XXXXX )X 0 0 0 0 0 0 0 0 0X
        XXXXX
        _____
        XXXXX 0
        XXXX0 X
        _______
          XXXXX 0
          XXXXXX
          ______
            XXXXX 0
            XXXXXX
            ______
              XXXXX0
              XXXXXX
              ______
                 XXXXX
                 XXXXX
                 _____
                     0
```

56. Another printer, under similar circumstances, used the letter O for zero, the letter 1 for one, and the letter X for all other numbers. Can you reconstruct the problem?

```
          XXXXX1
      _____________
XX1 ) 1O1O1O1O1
       X1X
       ___
        1XX1
        1XXX
        ____
          XXO
          XXX
          ___
           1XX1
           1XXX
           ____
             XXO
             X1X
             ___
              XX1
              XX1
              ___
                O
```

57.

```
      A B C
      A B C
      -----
    D E F C
  C E B H
E K K H
-----------
E A G F F C
```

58.

```
      A B C
      D E F
      -----
    G F D B
  C D H E
A K G A
-----------
A G B E G B
```

59. Ten cubes in stairstep:

```
x
x
x
xxxx
   x
   x
   xxxx
      x
      x
      xxxx
         x
         x
         xxxx
            x
            x
            xxxx
```

60.

```
           B E *
CAD ) * * A B * A
        C * *
        F E F *
        * * A *
          * * E *
          * * D F
              B E
```

61.

```
        6 * *
√ a b b c d c
  * *
    a * *
    * * *
      d * * *
      * * * *
            0
```

62. $PREM = P(MMPB)+1 = (EB)^2+(AM)^2$

What is the prime PREM?

63. BACC and CCCA are prime. AFEA and ACFB are squares of prime numbers.

64.

$$AAA \times BC = ADADA$$
$$BBB \times CA = CCBEE$$

65.

A	B	C	C
A	D	E	B
B	E	D	A
C	C	B	A

By replacing the letters with numbers, and reading right to left, up and down, left to right, and down and up, you obtain eight prime numbers, each used twice.

To save a long research in a table of primes, it is necessary only to know that the sum of the digits of the four numbers forming the sides of the square is 60, and the sum of the digits of the four numbers forming the interior square is 16.

66.

```
            Q U I
          P R E N D
          ---------
              * * *
          * * * *
          * * *
      * * *
      ---------------
CA )  * * * * * * * ( * * * * * *
      * *
      ---
        * * *
        * * *
        -----
          * * *
          * * *
          -----
              0
```

67.

```
      * B *
   ----------
 √  * * * * A
    *
    -
    * * *
      * *
    -----
        * * *
        * * *
        -----
```

68.

```
        * A B
   __________
  √ * * * * * *
    * A
    ---
      * * *
      * * B
      -----
      * * * * *
      * * * * *
      ---------
```

69.

```
          A A *
   ____________
  √ * * * * * A
    * *
    ---
    * * * *
    * * * *
    -------
      * * * * *
      * * * * *
      ---------
```

70. ABC, ACB, and ABC+ACB are the differences of the cubes of three consecutive numbers.

71.

```
                * * *
          ___________
  * * * ) A * * * 7 A
            * * 0 *
            -------
              * * * 7
              * * * *
              -------
                * * * *
                * * * *
                -------
                      0
```

The number represented by A occurs in the quotient, which is the same as the divisor.

72. KED = E × GB

KED is formed from three consecutive numbers; GB is formed from two consecutive numbers.

73. FAG = E × HBI

FAG is formed from three consecutive numbers.

74.

ABC = C^4
BCA = D^4 .

75.

```
          B O U L E
          R A D I S
          ---------
        1 . . . . 0
      1 1 . . . 0
    1 0 1 . . 0
    . . 0 . 0
  . . . . 0
  -----------------
  . . . . . . . . .
```

76.

```
        B A *
     ___________
   √ * * * * * A
     * A
     ---
     * * * *
     * * * C
     -------
         * * * *
         * * * *
         -------
```

77.

```
        TOCK
        TOCK
        ----
      *****
     *****
    *****
   *****
   ---------
   ****TOCK
```

78.

```
          R I V E
        F L A N C
        ---------
        x x x x 0
      x x x x 0
    1 x x x 0
  x 0 x x 0
1 x x 1 0
-----------------
x 1 x x 0 x x x 0
```

79.

```
   ..3.
    ..3
   ----
   3...
 ...33
 ....
-------
.......
```

80. GALON and ALONG are the squares of GOO and OOG to the base 6.

81. When the numbers 1234567890 are substituted for an English word, it is possible to make the following magic square. The sums of the rows, columns, and diagonals are 275.

RL	AA	CN	BD	RC
AE	MC	ML	EA	RB
UA	MB	EE	LR	AU
BR	EU	LM	LN	UE
BN	LD	NC	UU	BM

82. There are eight brothers and sisters in a family: Georgette, Ulysse, Yvon, Marie, Annette, Roger, Isadore, and Emile. All are less than 10 years old. If you represent the age of each child by the first letter of its name, you obtain

```
          YU
GUY )MARIE
     MARE
        EE
```

If Marie is the youngest of the sisters, what is her age?

83. All the digits and zero are used in the multiplier and the multiplicand. All the 2's in the problem are given. Find the two solutions.

```
         xxxxx
         xx2xx
        xxxx2x
       x2xx2x
      xxx2xx
     xxxx2x
    2xxxx2
(a) x2x22xxxxx
(b) x2xxxxxxxx
```

84. NXPSI × H = SPHINX

85.

```
DIOPHANTE
ETNAHPOID
---------
       TE
      NT
     AN
    HA
   PH
  OP
 IO
DI
D
```

86. What are the values of SOLFIER that correspond to the system of equations:

DO + RE = MI
FA + SI = LA
RE + SI + LA = SOL

87.

```
        ONE
     ------
TRY ) THIS*
      ***
      ---
       ***
       ***
       ---
       ****
       ****
       ----
```

88. In changing from the number system base 10 to base 9, VIN becomes EAU and OUI becomes NON.

89. Here is a bilingual:

(*a*) MAN + WOMAN = CHILD

M * AN * − WOMAN = CHILD

(*b*) HOMME + FEMME = * * T * ON

HOMME × FEMME = * * * * ENFANT

90.

```
        BOILEAU
        BOILEAU
        -------
      ****I***
     *******I
    *******L
   *******
  *******
 *******
 -------------
 *************
```

91. This problem is to the base 12:

```
       *9***
    --------
 ***)********
     ****
     ----
      ***
      ***
      ---
      ****
       ***
      ----
        ****
        ****
        ----
           0
```

92.

P × ABER = GLUT

P × ALGE = BRUT

93.

```
     J E A N            M A R I E
   M A R I E                J E A N
   ---------            -----------
     x x x x x          x A x x x x
   x x x x x          x A A x x x
 x x x x x          x x x A x x
x x A x A           -----------
-----------         x x x R L E T T E
x x x R L E T T E
```

94. All the digits are primes, different from 1:

```
  ***
   **
 ----
 ****
****
-----
*****
```

95. All the A's are shown:

```
               . . A .
         -------------
. A . ) . . . . A . .
        . . A A
        -------
          . . . A
            . . A
          -------
            . . . .
            . A . .
            -------
              . . . .
              . . . .
              -------
                    0
```

96.

```
          D E U X
          D E U X
          -------
        * * * * *
      X * * * *
    U * * * *
  E * * * *
  ---------------
  D U O S * * * *
```

97.

```
 M A I T R E
         × S
 -----------
 F I S T I E
```

98. TOC × TOC = ENTRE

99. This problem is to the base 12:

```
         C D E F
         C D E F
         -------
       G . . C F
     G . . . H
   . . . . L
 . . . . .
 ---------------
 . . . . . . C .
```

100.

```
    D E L L A
 + R I C C I A
 -------------
   A N G E L O
```

101.

```
 DO ) T H I S ( O N E
      * T
      ---
        * *
        * *
        ---
          I *
          * *
          ---
            0
```

102. The triangular number ABCD is divisible by 89, whereas DCBA is divisible by 11.

103. Here are four squares in triangular array vertically and horizontally. The largest is a multiple of 11.

```
xxxx
xxx
xx
x
```

104.

$$\begin{array}{rcl} ..\times.. &=& .77 \\ ..7\div.. &=& .7 \\ .\times.6 &=& .4 \\ ..-\ 9 &=& 1. \\ +\ \ .7+.. &=& .. \\ \hline .3.+.0 &=& 5.. \end{array}$$

105.

$$\begin{array}{r} x\ x\ A \\ A\ B\ x \\ \hline x\ x\ x\ D \\ x\ x\ D\ \ \\ x\ x\ B\ \ \ \ \\ \hline x\ x\ x\ x\ x\ x \end{array}$$

106.

$$\frac{\text{CENT}}{\text{I}} = \frac{\text{MINE}}{\text{CE}} = \text{CLE}$$

107. EAU × EAU = OCEAN

108. This is a magic square. All rows and columns add up to the same number.

MO	NE	DS	SN	TD
ID	NM	TE	ES	NE
SI	DO	NT	ID	ES
NS	MO	ID	TD	NO
IT	NM	TS	TT	DN

109. All ten digits are found in the multiplier and the multiplicand, and all the 1's are given.

```
       x1xxx
       xxxxx
      ------
      1xx1x1
     xxxxxx
     xx11x
   xxxxxx
  xxxx1x
  ----------
  xxxxxx11x1
```

110.

```
  (multiplicand)
   (multiplier)
  --------------
       34x7
      7xxx
  --------------
    (product)
```

111. All the 3's are given.

```
     **3*
      **3
     ----
     3***
   ***33
   ****
  -------
  *******
```

112.

```
       A B C
       D E F
       -----
     G F D B
   C D H E
 A K G A
 -----------
 A G B E G B
```

113. $$AB(A+B) = A^3+B^3$$

114.

$$\begin{array}{r} \text{CROSS} \\ \text{ROADS} \\ \hline \text{DANGER} \end{array}$$

115.

	(1)	(2)	(3)	(4)
(1)	*	*	*	
(2)	*	*	*	*
(3)			*	*
(4)			*	

Vertical:

(1) Pyramidal number (base 10)
(2) Prime (base 10)
(3) The square of N (base 8)
(4) Prime (base 10)

Horizontal:

(1) The square of N (base 9)
(2) The square of N (base 5)
(3) N (base 10)

116. Once in a while the letters of a cryptarithm will form the name of a number, and sometimes these numbers will form a mathematical truth. Like this:

$$\begin{array}{r} \text{THREE} \\ +\ \ \text{FOUR} \\ \hline \text{SEVEN} \end{array}$$

117.

```
  NINE
- FOUR
------
  FIVE
```

118.

```
  FORTY
    TEN
  + TEN
-------
  SIXTY
```

119.

```
  SEVEN
  SEVEN
+   SIX
-------
 TWENTY
```

120.

```
    ONE
    TWO
+  FOUR
-------
  SEVEN
```

121.

```
    ONE
    TWO
+  FIVE
-------
  EIGHT
```

122.

```
   F I V E
 - F O U R
 ---------
     O N E
 +   O N E
 ---------
     T W O
```

123. *Recreational Mathematics Magazine* reports a little game of poker played the other night. The card filling the winning hand is represented by the letter "A."

(A)(SPADE) = FLUSH

124. Here is another hand reported by the same magazine:

(A)(FLUSH) = TRUMPS

125. A puzzle correspondent sent me the following:

```
             W E Z A Z
       -----------------
 G U N ) B A Z O O K A
         G U N
         -----
         U E N O
         U B A A
         -------
           U U O O
           U U Z N
           -------
               U K K A
               U U Z N
               -------
                 W B N
```

If he had not made a mistake in copying the cryptogram, it would have been easy. Nevertheless, it is possible to find the incorrect letter, correct it, and solve the puzzle.

126. This one is so old that no one seems to know its origin:

$$\frac{EVE}{DID} = .TALKTALKTALK\ldots$$

127.

$$BABA = (AB)^2$$
$$AAAA = (CC)^2$$

This problem is to the base 7.

128.

```
          B A B A
BAB ) A A A B A A
      A A A
      B B A B
      A B A A
        A B A A
          A A B
          A A B A
          A A B A
                0
```

The digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are divided into two groups, A and B.

129.

```
        A x B x C
      x A x B x
    x x A x B
  x x x A x
x x x x A
-----------------
x x x x A x B x C
```

What multiplication will result in the above partial and general products?

130. *Recreational Mathematics Magazine* says the different species will mix:

```
   B E A V E R
+    T I G E R
   -----------
   R A B B I T
```

131.

```
        x x x x
        x x x x
        -------
      x x x x x
      x A x x
  x x B x x
  x x x x
  -------------
  x x x x x C x
```

A, B, and C are the only even numbers in the problem.

132.

```
   A H A H A
+    T E H E
   ---------
   T E H A W
```

133.

```
   xxx
    xx
   ----
  xxxx
 xxxx
 -----
 xxxxx
```

Every x is a prime greater than 1.

134. This is pure egoism. It isn't very good, but it is the first cryptarithm I ever devised.

```
          H S L E
      ___________
D C S ) S U D H U D E
        S D U H
        -------
          N G U
          D C S
          -----
          H I D D
          H U E L
          -------
            C G I E
            C G I E
            -------
```

135. This one is from *Recreational Mathematics Magazine*:

```
   T W O
 T H R E E
  S E V E N
 ---------
T W E L V E
```

136. Can you reverse a number by multiplying it by 4?

$$ABCDE \times 4 = EDCBA$$

No zero is used.

137. What is this square to the base 12?

```
        C D E F
        C D E F
        -------
      G * * C F
    G * * * H
  * * * * L
* * * * *
---------------
* * * * * * C *
```

138.

```
        xxxx                 xxxx
    ________             ________
xxxx)xxxxxxxx        xxxx)xxxxxxxx
     xxxx                 xxxx
     ____                 ____
     xxxxx                xxxxx
      xxxx                 xxxx
      _____                ____
        xxxxx                 xxxx
        xxxxx                 xxxx
        _____                 ____
            0                    0
```

The dividend is the product of the two different prime numbers which are the divisors. There are two possible solutions.

139.

```
  A B C D E
+ F G H K L
  _________
  E D C B A
```

Both 0 and 1 are contained in FGHKL, and A is greater than G.

140.

```
          * B * * *
          * * * * *
          _________
        A B * * * F
      B * * * * H
    C * * * * I
  D * * * * K
E * * * * L
_____________________
* * * * * * * * * *
```

All ten digits will be found in the multiplier and the multiplicand. They will also be found in the answer.

141.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
*	*	*	*	*	*	*	*	*	(I)
*	*	*	*	*	*	*	*	*	(II)
*	*	*	*	*	*	*	*	*	(III)
A	B	C	D	E	F	G	H	I	

(*a*) There are no zeros.

(*b*) Columns (1), (2), and (3); (4), (5), and (6); (7), (8), and (9); (1), (4), and (7); (2), (5), and (8); (3), (6), and (9) contain all the nine digits.

(*c*) The sums of (I) + (II), (II) + (III), and (I) + (II) + (III) each contain all nine digits.

142.

$$\begin{array}{r} \text{R E A D} \\ +\ \underline{\text{T H I S}} \\ \text{P A G E} \end{array} \qquad \begin{array}{r} \text{R E A D} \\ -\ \underline{\text{T H I S}} \\ \text{P A G E} \end{array}$$

These are two different problems. That is, the R in the addition and the R in the subtraction are two different numbers, and so on.

143.

$$\begin{array}{r} \text{x x x x x x} \\ \underline{\text{x x}} \\ \text{x x x x x x} \\ \underline{\text{x x x x x x}\ \ \ } \\ \text{O C T O B E R} \end{array}$$

where

```
O C T O
  C T O
  C T O B
    T O
    T O B E
    T O B E R
      O B E R
        B E R
        B E
          E R
            R
```

are 11 prime numbers.

144. Puzzlesmith J. A. H. Hunter received a letter from a reader referring to a "alphametical problem in which letters take the place of figures." It would seem that an *m* had been mistyped for a *b*. Mr. Hunter was struck with the possibilities of a new name "*alphametic*" to indicate puzzles in which real words were used.

The result was that in the October 27, 1955, issue of the Toronto *Globe & Mail* there appeared

FUN WITH FIGURES

by J. A. H. Hunter

It's just an easy alphametic today: merely a matter of commonsense and a little patience.

Each letter stands for a different figure, and you're asked to find the number represented by ABLE.

```
BE)ABLE(SIR
   MR
   --
   RRL
   RLM
   ---
     BE
     BE
     --
```

So a new name was born. The following eight puzzles are from Mr. Hunter's fertile brain.

145.

```
  N E V E R
– D R I V E
  ---------
    R I D E
```

146.

```
F I T ) P O L E ( T O
        L O T
        -----
        F R E E
        F R E E
        -------
```

147.

```
   G E T
     O N
   -----
   R O N
 G E T
 -------
 G R A N
```

148.

```
  S T A R T
  S T A R T
    T H I S
  S T A R T
-----------
A R I G H T
```

149.

```
D I D ) J U D G E ( I T
        x x x x
        -------
          I D L E
          I D L E
          -------
```

150.

```
   T I E
     I T
  -------
  S A V E
S P A R
---------
S T E V E
```

151.

```
   P E N I T E N T
 - S P I N S T E R
   ---------------
     R E P I N E D
```

152.

```
W A S ) M A S K E D ( W A S
        x x x x
        -------
          x x x x
          x x x x
          -------
            x x x x
            A x x x
            -------
              S P Y
```

153. Here is a new form of cryptarithm. Each digit can be represented by two letters, but each letter represents only one digit.

```
      √‾D O U B L E‾ ( S I R
         X
H I    | P U B
       | L E T
       ---------
H Y R     | G A L E
          | G A L E
          ---------
```

154. This problem started life as a pictorial cryptarithm, but ended up like this:

```
          _________
    B E ) A B L E (M A N
          M N
          ---
            L L
            A T
            ---
              H E
              H E
              ---
```

155.

$$\begin{array}{r} YOU \\ I^2 \\ \hline aaa \\ aaaa \\ \hline THaNK \end{array}$$

156. And finally, an anti-cryptarithm:

Prove $NA \cdot CL \neq SALT$ to the base 6.

ANSWERS

ANSWERS

1. (*a*) In the second partial product we see $D \times A = D$, hence $A = 1$.

(*b*) $D \times C$ and $E \times C$ both end in C, hence $C = 5$.

(*c*) D and E must be odd. Since both partial products have only three digits, neither can be 9. This leaves only 3 and 7. In the first partial product $E \times B$ is a number of two digits while in the second partial product $D \times B$ is a number of only one digit. Thus E is larger than D, so $E = 7$ and $D = 3$.

(*d*) Since $D \times B$ has only one digit, B must be 3 or less. The only two possibilities are 0 and 2. B cannot be zero because 7B is a two-digit number. Thus $B = 2$.

(*e*) By completing the multiplication, $F = 8$, $E = 7$, and $G = 6$.

(*f*) The answer is $125 \times 37 = 4625$.

2. (*a*) T must equal 1 and $W \leqq 4$, since THREE contains five digits.

(*b*) $E = 4$ because 44 is the only combination of two equal digits that terminate a square. Hence $O = 2$ or 8.

(*c*) The answer is $TWO = 138$.

3. There are 22 ways of satisfying the relation $I \times RL = TS$ if we make the obvious restrictions $I \neq 1, 5, 9$ and $L, S \neq 0, 1, 5$. The condition $L \times TS = BTO$ narrows the number of possible solutions to 6. The condition $E \times BTO = RTMP$ gives the unique solution:

5	1960
4	392
7	98
	14

So:

0	1	2	3	4	5	6	7	8	9
P	R	O	B	L	E	M	I	S	T

4. This is Berwick's solution of his own problem:

(*a*) Since the multiplication of the divisor by 7 gives 6 figures in the product, while the second and fourth multiplications give 7 figures, the divisor must begin with 11, 12, 13, or 14, and the second and fourth figures of the quotient are 8 or 9.

(*b*) Since the product of the divisor multiplied by 7 has as its second digit 7, it is found by trial that the divisor must begin with 111, 124, 125, 138, or 139.

(*c*) The third remainder must evidently begin with 10, and therefore also the fourth product begins with 10. Hence either the divisor begins with 111 and the fourth figure of the quotient is 9, or the divisor begins with 125 and the fourth figure of the quotient is 8.

(*d*) But if 9 is the fourth figure of the quotient, then, since the product has as its third digit from the right 7, the divisor would have to begin with 11197, which would make the product by 7 have an 8 in the second place instead of a 7. Hence the fourth figure in the quotient is 8, and the divisor begins with 12547, the sixth figure being less than 5.

(*e*) Since the third product (by 7) begins with 878, and the line above cannot be greater than 979. . ., therefore the remainder begins 101; and since the fourth product begins 100, therefore the last remainder begins with a 1, and the last figure of the quotient must be 1.

(*f*) On trying 4, 3, 2, 1 in succession as the sixth digit in the divisor, and working the division backwards, it is found that the second of these alone satisfies the conditions not yet considered, and that the following is the full division:

```
125473)7375428413(58781
        627365
        -------
        1101778
        1003784
        -------
          979944
          878311
          -------
          1016331
          1003784
          -------
            125473
            125473
            ------
```

5. (*a*) None of the digits is 0 because each gives a product.

(*b*) Since $A \times C$ ends in A and $B \times C$ ends in B, C must be either 1 or 6. But since the first product contains four digits, $C > 1$, hence $C = 6$.

(*c*) A and B are even (but not 6), hence A and B are 2, 4, or 8. Now, A cannot be 4 or 8 since the second product contains three digits, hence A is 2.

(*d*) B must be 4 or 8. B cannot be 4 because the last product contains four digits. Hence B is 8.

(*e*) The multiplication is 286×826.

6. (*a*) The square of D does not end in D, hence D cannot be 0, 1, 5, or 6. C must equal 1, 4, or 9. Thus the number sought must terminate in one of the combinations 42, 93, 64, 97, 48, 19.

(*b*) $CD \times D$ terminates with DC, and the only groups meeting this condition are 97 or 48. $D \times B$ terminates in C. Assume $CD = 97$; then $B = 7 = D$, which is impossible.

(*c*) Thus $CD = 48$, which means B must equal 3 or 8. But since $D = 8$, B must equal 3 and $BCD = 348$. Since $BCD \times D$ terminates in ADC, A must equal 7, and $ABCD = 7348$.

7. (*a*) None of the letters A, B, C, D, or E are zero.

(*b*) A and C are smaller than B, D, or E because their products contains only five digits.

(*c*) $A < 3$ because if $A = 3$, B is at least 4 and 3×34000 is greater than 100,000.

(*d*) We next prove that $E = 7$ since E is greater than A or C. E is not 1, 2, or 3 (the fourth product contains six digits), nor is it 5 or 6 because in those cases $D = E$. Therefore $E = 4, 7, 8$, or 9. The only value leading to a solution is 7. This means $D = 9$.

(*e*) A must be equal to either 1 or 2. But it can be shown that A cannot equal 2. Thus the multiplication is 18497×47981.

8. Since the remainder of the division is zero, the product $I \times I$ terminates in E, a digit different from I, which means I cannot equal 0, 1, 5, or 6.

The product $QUI \times C$ terminates in E. Thus C and I, two different digits, when multiplied by I, give products ending in E.

Therefore I cannot be 1, 3, 7, or 9. This leaves only three values for I, each of which establishes values for E and C.

I	E	C	C E C I
8	4	3	3 4 3 8
4	6	9	9 6 9 4
2	4	7	7 4 7 2

The first hypothesis requires that QUI × 8 be a number of three digits, so QUI < 125. Since U must be different from Q, the only number satisfying this condition is 108. But this makes T = 3 = C, which is impossible.

The second hypothesis indicates that T > C, whereas C = 9, again impossible.

Thus CECI must equal 7472. QUI × 7 is less than 1000, and QUI $\leqq$ 142. Since I = 2 and Q and U are different, QUI must equal 102 or 132. But if QUI = 102, T = 7 = C. Hence QUI = 132.

So the answer is 132 × 7472 = 986304.

9. G is neither 0 nor 1, and A and F are larger than G. If G $\geqq$ 3, the multiplicand is $\leqq$33333 and A $\leqq$ 3, but A > G. Hence G = 2 and A = 3 or 4. If A = 3, BB = 66 or 77, but BB ÷ G = AB which must be 33 or 38, either of which is impossible. Thus A = 4 and B = 8 or 9. Set BB = 88 and AB would become 44. This being impossible, B = 9.

C × G > 10, hence C $\geqq$ 5. ABC must be 495, 496, 497, or 498; the first partial product (**H***) begins with 198 or 199. Since B = 9, H must equal 8.

The digit F in the multiplier cannot be 3, 5, 6, or 7. But AB × F increased by the carry-over must be F, making F = 5. Carrying over 1, the fourth digit of the third partial product must be 0 or 1. Now, 2480 or 2481 divided by F gives 496 for ABC, hence C = 6. Finally the sixth digit of the result is D, which means E = 7 and D = 0.

The answer is 49607 × 524 = 25994068.

10. A, the last number of the square, must be 0, 1, 4, 5, 6 or 9. The square of A contains two digits, which excludes 0 and 1.

The number 9 is also excluded because the partial product contains three digits and $189 \times 9 = 1701$. The possible remaining numbers are 442, 448, 555, 664, or 666. The last partial product begins with A, making 555 the only choice for the square root.

11. (*a*) P = 1.

(*b*) L^2 ends in 6 and must be 4 or 6.

(*c*) $S \times L$ equals a number ending in S. No multiple of 4 ends in that same number, so L = 6 and S = 2, 4, or 8.

(*d*) $(L \times I) + 3$ ends in 1, so I must be 3 or 8. If I is 3 and S is 8, E must also be 8, which is impossible. If I is either 3 or 8, and S is 4, E must also be 4, which is impossible. Hence S = 2 and E = 7 (whether I = 3 or 8).

(*e*) Either 2A or 2A+1 equals a number ending in A. The only possibility is A = 9.

(*f*) 2R equals a number ending in U, which must make U equal 8 or 0.

(*g*) 2V equals a number beginning with 1, hence V must equal 5 or 8.

(*h*) By elimination, R = 4. This in turn makes U = 8, which makes I = 3 and V = 5.

$$15460632 \div 95436 = 162$$

12. (*a*) A^2 is either 9 or a two-digit number, so A must be 3 or greater.

(*b*) $R \times A$ is a one-digit number and R is not 1, hence R must be 2 or 3.

(*c*) If R is 3, A must be 2. This is contrary to argument (*a*). Hence R = 2.

(*d*) A must be 3 or 4 in order that 2A be a one-digit number. But if A is 3, the third partial product will be a four-digit number; hence A = 4, making M = 8.

(*e*) S must be an even number, hence it is either 0 or 6. If 0, the last number in the multiplicand must be zero which would make all the last numbers in the partial products be zero. Since they are all different, S must be 6.

(*f*) 4213×2486 is the number.

13. 6×28 = SIX = 168. The first digit of each 3-digit partial product must be 1 or 2. The first digit of the fourth partial product must be 2. So the fourth digit of the quotient is 8 (the third and fifth are zero). This establishes R as being 2 and V as being 0. The fourth partial product begins and ends with 2, making the sixth digit of the quotient 9. This establishes E as 5, which in turn establishes F as 4.

$$4502652 \div 28 = 160809$$

14. (*a*) A is smaller than 4.

(*b*) A and B are smaller than C, D, or E.

(*c*) A, B, C, D, and E are not zero.

Assume E is even. Then A, D, G, and F must be even and A = 2. D is not zero, therefore either G or F must be zero. Thus either D or B is 5. But D is even and if F is 5, the second product will have six digits.

Therefore E is odd. By elimination:

$$ABCDE = 24697$$

15. (*a*) E is either 4 or 6.

(*b*) M is double E and is a single digit, so E = 4 and M = 8.

(*c*) From the last partial product, S = 0, I = 9, and L = 2.

(*d*) The last digit of the second partial product is 0, so U = 5.

(*e*) From here, it can be successively proved that R = 7, O = 3, A = 1, and V = 6.

16. An examination of the partial products leads to these conclusions:

(*a*) The eight digits in the multiplier are all different.

(*b*) 0 and 1 are not in the multiplier.

(*c*) Therefore the sums of the multiplier and multiplicand are 44.

(*d*) The first digit of the multiplicand is either 8 or 9.

(*e*) The last digit of the multiplicand is 1, 3, 7, or 9. Examination will show that 1 and 9 are impossibilities.

(*f*) N must equal 9 because none of the partial products begin with N.

These facts limit the combinations to the unique solution 9188837×75436928.

17. Let the quotient be A00B0CD and the divisor be N. Then

$$499 \leqq A \times N \leqq 598$$
$$995 \leqq B \times N \leqq 999$$

$995 = 5 \times 199$, $996 = 3 \times 4 \times 83$, 997 is prime, $998 = 2 \times 499$, and $999 = 3 \times 9 \times 37$.

The only possible factor is $N = 199$ and the quotient is 3005028.

18. (*a*) The second partial product ends in zero, and $2E < 10$. So $T = 5$ and E is even (2 or 4). Since $E \times E{*}{*}{*}T = T{*}{*}{*}{*}$, E must equal 2.

(*b*) The left-hand digit of the multiplier is 1.

(*c*) $P = 4$.

(*d*) All the T's are given, so the digit on the right of the third partial product is zero, and the second digit from the left of the multiplier is 8.

(*e*) The multiplier is $1825 = 5^2 \times 73$.

(*f*) Because PIGEOLET is a cube, it is the cube of 5×73.

19. (*a*)
$$10 + E - S = M$$
$$S - E = M$$
$$2M = 10$$
$$M = 5$$

(*b*) $2E + 1$ ends in 5, so E is either 2 or 7. $S - E = 5$, so $E = 2$ and $S = 7$.

(*c*) $B = 9$.

(*d*) $5 + T + 1 = R$, a two-digit number. So $T = 4$, 6, or 8. $R > 2$ and since $E + R = E$, $R = 0$ and $T = 4$.

(*e*) $P = 1$.

(*f*) SEPTEMBRE = 721425902.

20. R must equal zero. $2U + F \leqq 9$, hence $U = 1$, 2, 3, or 4. And USA = 178, FDR = 230, and NRA = 408.

21. (*a*) $B \times D$ and $C \times D$ give two-digit numbers ending in D. This is possible only if $D = 5$. B and C must equal 3, 7, or 9.

(*b*) If $B = 3$, $*D = 15$; if $B = 7$, $*D = 35$; if $B = 9$, $*D = 45$.

(*c*) If $C = 3$, ACD = 105 or 135; if $C = 7$, ACD = 135 or

315; if C = 9, ACD = 135 or 315. Only 135 is compatible, so C = 3 and B = 9.

(*d*) The star in *C must be an even number and give a two-digit number beginning with 1 when multiplied by 4. Only 4 meets these conditions.

(*e*) $5 \times 9 \times 43 = 1935$.

22. (*a*) $D+F-E = 0$ or 11.

(*b*) $2F+E$ terminates in D, hence

$$F = E-D(+0 \text{ or } +11)$$
$$2F = D-E(+0,\ +10, \text{ or } +20)$$

and $$3F = 0, 10, 11, 20, 21, \text{ or } 31.$$

Only 21 fits, and F = 7.

(*c*) Thus: $D = 4+E$

and $E+A+1 = 10$

or $F+E-A = 0$ or 11

E = 1, A = 8, D = 5.

(*d*) The cube is $803^3 = 517781627$.

23. The square of TIF ends in TIF and must be 625 or 376. Since N = 1, only TIF = 625 will satisfy. The remaining numbers are obtained by multiplication: A = 0, G = 9, S = 4, E = 3, O = 8, and P = 7.

24. (*a*) E is not 1 and since $E \times E$ terminates in E, it must be 5.

(*b*) The partial product $VALSE \times L$ contains five digits, so L = 1 or 2. But $E \times (S+V)+2$ ends in L, making L = 2, so S = 1.

(*c*) $VALSE \times 2$ is smaller than 100,000, making V = 3 and leaving A = 7.

(*d*) 37215×12735.

25. The last partial remainder ends in 000 and the next-to-last remainder ends in 0. The divisor does not end in zero, otherwise the next-to-last partial product would end in zero and the rest would be zero. The divisor is a multiple of 5, the next-to-last final product ends in 5 and the last partial product is 5000. The last digit of the quotient is 8 and the divisor is 625. None of the other partial products exceed three digits, so the quotient is 1011.1008.

26. The difference between the squares of two consecutive numbers is equal to the sum of the numbers, hence:

$$\text{OMLI} - \text{OHEM} = \text{LET}$$

But RG + RA = LET. Hence L = 1.

The subtraction gives E = 5 or zero.

If E = 5, R = 7, which is impossible since the sum of the squares of two consecutive numbers cannot end in 1, 3, or 5. Thus E = zero and R = 5 while O = 2.

The sum of A and G is less than 10, so G = 3 and A = 4.

The key is LOGARITHME.

27. (*a*) The first equation indicates that R^2 has two digits. Since R is prime, it must be 5 or 7. If R = 5, the second equation indicates A = zero (impossible) or A = 5 = R (impossible). Hence R = 7.

(*b*) 49 × NIG terminates in 7, so G = 3.

(*c*) The other numbers are A = 1, N = 2, L = 4, E = 5, D = 6, O = 8, I = 9, T = zero.

28. Since each letter is a final digit, they must all be odd: 1, 3, 7, or 9.

A and C must be 1 or 7, otherwise the numbers ADDD and AACA would be divisible by 3.

Thus B and D must be 3 or 9.

Under these conditions, BCDB can be written in four ways:

B = 3	D = 9	A = 7	C = 1	3193, divisible by 31
B = 3	D = 9	A = 1	C = 7	3793, prime
B = 9	D = 3	A = 7	C = 1	9139, divisible by 13
B = 9	D = 3	A = 1	C = 7	9739, prime

In both cases where a prime results, C = 7 and A = 1.

BDAC must be either 9317 or 3917 as B and D equal 9 and 3.

But 9317 is divisible by 7 whereas 3917 is prime.

Thus A = 1, B = 3, C = 7, and D = 9.

29. (*a*) The four digits are all different, else there would not be 24 different permutations.

(*b*) Since there are 24 permutations, 12 odd and 12 even, then two digits are odd and two are even.

(*c*) One of the odd digits must be 1 or 9 because the square of a prime must end in one of these numbers; however, the odd digit cannot be 5 because a prime cannot end in 5.

(*d*) The only combination of the two terminal digits of a four-digit number divisible by 4 are 04, 08, or 48. 08 or 80 following the odd digits are not divisible by 8; this eliminates 0 and the even digits must be 4 and 8.

(*e*) The divisibility by 16 and the impossibility of division by 3 eliminates most combinations.

(*f*) Finally, the combination 1348 is the solution.

(*g*) The four primes are 1483, 4813, 4831, and 8431.

The seven products:

$47 \times 89 = 4183$ $19 \times 97 = 1843$ $17 \times 479 = 8143$
$47 \times 179 = 8413$ $13 \times 337 = 4381$ $23 \times 167 = 3841$
$19 \times 439 = 8341$

The square of a prime is $59^2 = 3481$.

Eight multiples of 2: 1438, 4138, 1834, 8134, 3418, 4318, 3814, 8314.

Two multiples of 4: 1348, 3148.

One multiple of 8: 1384.

One multiple of 16: 3184.

30. The square of C ends in C, thus C must be 0, 1, 5, or 6. Since ABC is prime, C = 1. For the square BC to end in BC, B must equal zero. ABC can be 101, 401, 601, or 701. And only 601 is prime.

31. C = 0 and G = 1. J = H+I, A = 2, B = 4. Thus:

```
240)893:
    720:
    ----
    173:
    168:
    ----
      5:
      4:
```

32. Let $1000 \cdot O + 100 \cdot D + 10 \cdot E + R$
$$= 18(10 \cdot D + O + 10 \cdot D + R).$$

Then $802 \cdot O + 10 \cdot E = 80D + 17R$.
Or (mod 10) $2 \cdot O = 7 \cdot R$.
From which $R = 2, 4, 6,$ or 8 and $O = 7, 9, 1,$ or 3.
Only the hypothesis $R = 6$ and $O = 1$ gives the solution:

$$ODER = 1926 = 18(91 + 16)$$

33. It can be established without difficulty that the third digit of each factor is zero, that the product is in excess of 10 million, and that the two first digits of one factor are equal and are less than the last digit. This factor (the quotient of the division on the left) must be 1103, 1109, or 3307. The other factor is 9803 or 9901 when the first is 1103; 9203, 9209, 9403, 9601, 9803, or 9901 when the first is 1103; 3301 when the first is 3307. There are nine solutions in all.

34. O and L must be 1 and 2 or 2 and 1. Hence MASURE must equal 354879, 354978, 793845, or 793548. OIL = 162.

35. (*a*) A = 1 with T = zero or 9.
(*b*) If T = zero, E must equal 9, making L = T = zero, which is impossible. Hence T = 9, E = 8, L = zero, and R = 2.
(*c*) Since REEL/TIL is a whole number, the complete solution is ACIDE = 13658, ETHER = 89782, and ALCOOL = 103440.

36. Let N = ABCBCACAB.
(*a*) N is divisible by 11 since DD is a multiple of 11; $3C - (A + B)$ is a multiple of 11 (from the old rule that if alternate addition and subtraction of the digits gives a number divisible by 11, the number itself is divisible by 11: $A - B + C - B + C - A + C - A + B = 3C - (A + B)$).
(*b*) N = multiple of $111 + ABC + BCA + CAB$
= multiple of $111 + 111 \times (A + B + C)$
= multiple of 111 = multiple of 3×37.

Experts will tell anyone who is puzzled by the 111 that it is an inherent property of numbers that if they are marked off in threes from the left and each group is subtracted from the original number, the result is divisible by 111.

(*c*) Hence one of the factors is 37 or 74 and D = 3, 4, or 7.

(*d*) If all five factors are given their maximum values, the products obtained give the only possible values of A to be 1 or 2.

(*e*) From these facts, the unique solution is ABC = 152.

37. PL**UIE divided by PLIC gives PLOC only if PL = 10. There are three solutions:

$$1034 \times 1054,\ \ 1073 \times 1023,\ \text{and}\ \ 1028 \times 1058$$

38. The factors 2 and 5 are eliminated reciprocally because in this case C would equal zero. Two cases remain: C = 2 or 5, with A, B, and D being 1, 3, or 7. The only solution is 77175.

39. $a = 1$ can be rejected.
$a = 2$ gives: $2^b c^2 = 2bc2$.

Set up the first nine powers of 2 (I) and the squares of the first nine numbers (II). The product of a number from (I) and a number from (II) must end in 2 and have four digits.

$$2^5 \cdot 9^2 = 2592$$

For $a = 3$, the first eight powers of 3 (four digits maximum) and the cubes of the first nine numbers must give $3bc3$.

There is no such solution.

40. (*a*) Neither the multiplier nor the multiplicand contains zero. S = 1 (six different products of six digits each).

(*b*) 2P is a one-digit number (from its final product) so P must be 2, 3, or 4.

(*c*) The digit in the 100,000 column of the final product is 6S. Since S = 1, I = 6 + the carry-over of 5P.

(*d*) The carry-over is 1 or 2, hence I = 7 or 8.

(*e*) Let k be the second digit from the left in the multiplier. $k \times X$ terminates in 1.

1×1 is impossible.

9×9 is impossible because there are only six digits in the partial product.

7×3 is impossible because $P \leqq 4$.

3×7 is the remaining possibility, so X = 7; hence I = 8 and H = 2.

(*f*) The third digit from the right in the product, N, must equal 4 or 5. But 4 is impossible, otherwise P (in the product) would be greater than 4. Thus N $=$ 5 and P $=$ 4.

(*g*) SPHINX represents the period of 1/7.

41. Let B be the upside-down multiplication and A be the conventional multiplication.

(*a*) In A, the first partial product contains four digits, so H $=$ 1, 2, or 3. If H $=$ 2, X $=$ 0 or if H $=$ 3, X $=$ 5 and I $=$ 1. Both are impossible because of B's partial products. So H $=$ 1.

(*b*) In B, the fourth partial product has five digits, which makes X greater than 5 (since H $=$ 1).

(*c*) Further, in A, the third partial product contains $X(I-1)$, which is a multiple of 10. This makes I $=$ 6 and X $=$ 8.

(*d*) The other digits follow: Z $=$ 9, O $=$ 7, and S $=$ 3.

42. These relations are established;

$$\text{NED}+\text{SASH} = 2\text{SHUN} \quad (1)$$
$$\text{NED}\cdot\text{SASH} = \text{SHUN}\cdot\text{SEED} = (\text{SEND})^2 \quad (2)$$
$$\text{SASH} > \text{SHUN} > \text{SEND} > \text{SEED} > \text{NED} \quad (3)$$

(*a*) From (1), S $=$ 1.

(*b*) From (2), $D\cdot H \equiv D^2 \pmod{10}$ and $D\cdot N \equiv D^2 \pmod{10}$. Thus D $=$ 0 or 5.

(*c*) From (1), D and H have the same parity with $H < 5$.

(*d*) From (3), $A > H > E$.

(*e*) If D $=$ 0, $ED\cdot SH \equiv 0 \pmod{100}$, hence $E\cdot SH \equiv 0 \pmod{10}$; H must be even, making E $=$ 5. But from (*d*) $H > E$, contradicting (*c*). Therefore D $=$ 5; H, being odd, $=$ 3; and E, being smaller than H, $=$ 2.

(*f*) From (*b*) N must be odd and as $D+H = 8$, N must equal 9.

(*g*) $ED+SH = 25+13 = 38$, hence UN $=$ 19 or 69. But U $=$ 1 is untrue, thus N $=$ 9.

(*h*) Hence NED $=$ 925, SASH $=$ 1813, SHUN $=$ 1369, SEED $=$ 1225, and SEND $=$ 1295.

43. Since the multiplicand contains all 10 digits, it is divisible by 9. Also, the sum of the digits of the product, $9T+2S$, is divisible by 9. Thus S must equal 9 or zero. Only the value S = zero will give the solution:

$$8641975230 \times 9 = 77777777070$$

44. (*a*) There are 10 letters. All ten occur in the units and tens columns, but only nine occur in the hundreds column. The non-occurring one is E, which must be zero.

(*b*) Eight of the nine letters in the hundreds column occur three or four times. Only C occurs twice. Hence it must be 1 or 9.

(*c*) If C = 9 and A = 1 (the smallest number possible), CEL × AA would equal at least 9900. However, the sum of four numbers, each less than 999, is less than 9900. Hence C must equal 1.

(*d*) The only reasonable number for CEL × AA would call for A to equal 2.

(*e*) AEA must equal 202 and AAG = 22G. The only value for G that will fit is 9. The difference in the arithmetical progression is 27.

(*f*) OGG = O99; less 27, we have O72, which is OFA and F = 7. O72 less 27 = O45 which is OOR: O = 4 and R = 5. 445 less 27 = 418 which is OCU; and U = 8.

(*g*) LCE = L10. L10 + 27 = L37, making L = 3 and I = 6.

45. If the numbers are written in the decimal system

$$(\mathrm{E}+8\mathrm{N}+64\mathrm{N}+512\mathrm{A})-(\mathrm{E}+5\mathrm{N}+25\mathrm{N}+125\mathrm{A}) = \mathrm{E}+7\mathrm{N}+49\mathrm{N}+343\mathrm{A}$$

or 44A − 14N = E.

A, N, and E are less than 5 and E is even. The only solution is ANNE = 1332.

46.

$$\frac{B^2}{2}-B-\left(\frac{A^2}{2}-A\right) = 12A$$

$$B^2-2B-(A^2+22A) = 0$$

The discriminant $(4A^2+88A+4)$ is a perfect square.

$$A = 2 \quad \text{and} \quad B = 8$$

or

$$A = 6 \quad \text{and} \quad B = 14$$

* * *

At this point we will cease leading you by the hand. The answers to the rest of the problems will be given, but the method of finding them will be up to you.

47. $29705+6475 = 36180$

48. 14523×89076

49. $38712+21783 = 60495$

50. 126984×7503

51. $\dfrac{82112}{13069} = 3.14147 \times 2$

52. $122023936 = 496^3$

53. $222222222222 \div 900991 = 246642$

54. $12128316 \div 124 = 97809$

55. $10000000001 \div 27961 = 357641$

56. $101010101 \div 271 = 372731$

57. $685^2 = 469225$

58. 783×956

59.
4
9
1
3375
8
3
2304
0
9
6589
2
6
1331
7
2
8000

60. $103123 \div 537$

61. $632^2 = 399424$

62. PREM $= 4561 = 4(1140)+1 = 60^2+31^2$

63. BACC $= 9133$; CCCA $= 3331$
AFEA $= 1681 = 41^2$
ACFB $= 1369 = 37^2$

64. $222 \times 91 = 20202$
$999 \times 12 = 11988$

65.

1	9	3	3
1	4	0	9
9	0	4	1
3	3	9	1

66. $(168 \times 30275) \div 49$

67. $20449 = 143^2$

68. $715716 = 846^2$

69. $994009 = 997^2$

70. $271 = 10^3 - 9^3$
$217 = 9^3 - 8^3$
$271 + 217 = 10^3 - 8^3$

71. $682276 \div 826 = 826$

72. $234 = 3 \times 78$

73. $189 \times 3 = 567$

74. $625 = 5^4$; $256 = 4^4$

75. 14530×62789

76. $793^2 = 628849$

77. TOCK $= 9376$

78. 6170×35284

79. 1237×893

80. GOO = 122; OOG = 221

81. CUMBERLAND

82. Three

83. 89104×36275
65103×49287

84. SPHINX = 163854

85. DIOPHANTE = 123456789

86. SOLFIER = 1487065 or 1354076

87. 2 9870 = 1 4 5 × 2 0 6
THIS* = ONE × TRY

88. VIN = 298 and EAU = 361
OUI = 719 and NON = 878

89. MAN = 856, WOMAN = 39856
HOMME = 81557, FEMME = 67557

90. 3120584

91. $12150518 \div 124$

92. $4 \times 1738 = 6952$
$4 \times 1963 = 7852$

93. JEAN = 7206
MARIE = 50432

94. 775×33

95. $3527876 = 484 \times 7289$

96. 9876×9876

97. 124390×6

98. 208^2

99. 6789^2

100. $52008 + 791198 = 843206$

101. $14 \times 457 = 6398$

102. ABCD = 3916

103. 7744
729
49
4

104.

$$\begin{array}{r} 13 \times 29 = 377 \\ 357 \div 21 = 17 \\ 4 \times 16 = 64 \\ 19 - 9 = 10 \\ 37 + 15 = 52 \\ \hline 430 + 90 = 520 \end{array}$$

105. 118×849

106. $\frac{1674}{9} = \frac{2976}{16} = 186$

107. 203^2

108.

M	E	D	I	T	O	N	S
1	2	3	4	5	6	7	8

109. 41057×96283

110. 1139×73

111. 1237×893

112. 783×956

113. AB = 37 or 48

114. $96233 + 62513 = 158746$

115. 831
4101
23
1

116.

$$\begin{array}{r} 79422 \\ 3104 \\ \hline 82526 \end{array} \quad \text{or} \quad \begin{array}{r} 28566 \\ 7495 \\ \hline 36061 \end{array} \quad \text{or} \quad \begin{array}{r} 36255 \\ 8902 \\ \hline 45157 \end{array}$$

117.

```
 5456
 2980
 ----
 2476
```

118.

```
 29786
   850
   850
 -----
 31486
```

119.

```
  68782
  68782
    650
 ------
 138214
```

120.

```
   530
   625
  9548
 -----
 10703
```

121. N, W, V = 2, 4, 7 in any order

```
     6 N 1
     8 W 1
   9 0 V 6
 ---------
 1 0 5 3 8
```

122.

```
 3496     9516
 3210     9280
 ----     ----
  286      236
  286      236
  ---      ---
  572      472
```

123. (5) (13582) = 67910. Since the statement given says that the winning hand (a FLUSH in this case) was filled by a card represented by A, then the total of six letters in A FLUSH must represent only five cards, and therefore one of the letters in FLUSH must be a 1 (A cannot be 1). The only other solution fulfilling this condition is (4) (17453) = 69812, but this would mean indicating a Queen by the notation 12 which is seldom done.

124. (6) (45183) = 271098

125. Subtracting GUN from BAZ in the first partial product should be UEU instead of UEN. This gives us

$$6079940 \div 325 = 18707$$

126. $\dfrac{212}{606} = .349834983498\ldots$

$\dfrac{242}{303} = .798679867986\ldots$

127. BABA = 2424 = 42^2
AAAA = 4444 = 55^2

128. $774288 \div 283 = 2736$

129. 12257×64512
6254×36524
or 6254×86524

130. $251453 + 60753 = 312206$

131. 1937×1937

132. $47474 + 5272 = 52746$

133. $775 \times 33 = 25575$

134.

S	C	H	E	D	U	L	I	N	G
1	2	3	4	5	6	7	8	9	0

135.

$$\begin{array}{r} 106 \\ 19722 \\ 82524 \\ \hline 102352 \end{array}$$

136. $21978 \times 4 = 87912$

137. 6789×6789

138. 9907×1103
9907×1109

139. $59427 + 13068 = 72495$

140. 80679×34215

141.
```
183724578
497651346
265839912
---------
947215836
```

142. $3285+1607 = 4892$
$5809-3741 = 2068$

143.

O	C	T	O	B	E	R
9	8	2	9	1	7	3

144. $9207 \div 27 = 341$

145. $93536-86753 = 6783$

146. $9782 \div 146 = 67$

147. $406 \times 12 = 4872$

148.
```
 62182
 62182
  2496
 62182
------
189042
```

149. $16492 \div 434 = 38$

150. $658 \times 56 = 36848$

151.
```
72418248
67146825
--------
 5271423
```

152. $523048 \div 723$

153. 345^2

154. 12)4152(346

155.
```
  209        236        249
   8²         8²         8²
  ---        ---        ---
  aaa        aaa        aaa
 aaaa       aaaa       aaaa
-----      -----      -----
13a76      15a04      15a36
```

156. Set up addition and multiplication tables:

+	1	2	3	4	5
1	2	3	4	5	10
2	3	4	5	10	11
3	4	5	10	11	12
4	5	10	11	12	13
5	10	11	12	13	14

×	1	2	3	4	5
1	1	2	3	4	5
2	2	4	10	12	14
3	3	10	13	20	23
4	4	12	20	24	32
5	5	14	23	32	41

(*a*) A·L is a number ending in T, so A and L are not 0 or 1.

(*b*) S is not 0 or it would not be the first digit of a four-digit number.

(*c*) C·N is a two-digit number. If one of them is 1, then S must be 1, which is a contradiction, so C and N are not 0 or 1.

(*d*) By elimination, T must be 0 and S must be 1.

(*e*) Since T = 0, A and L must be 3 and 4 or 2 and 3.

(*f*) By elimination, N and C must be 4 and 5 or 2 and 3.

(*g*) But $5 \times 4 = 32$, which would make S either 3 or 4, hence N and C must be 2 and 5. This in turn eliminates A or L as being 2.

(*h*) This leaves four possibilities:

$$\begin{array}{r} 24 \\ \underline{53} \\ 120 \\ \underline{212} \\ 2240 \end{array} \qquad \begin{array}{r} 23 \\ \underline{54} \\ 140 \\ \underline{203} \\ 2210 \end{array} \qquad \begin{array}{r} 53 \\ \underline{24} \\ 340 \\ \underline{150} \\ 2240 \end{array} \qquad \begin{array}{r} 54 \\ \underline{23} \\ 250 \\ \underline{152} \\ 2210 \end{array}$$

None of these satisfies the original equation.